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Variable input observer for state estimation of high-rate dynamics

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Abstract

High-rate systems operating in the 10 μs to 10 ms timescale are likely to experience damaging effects due to rapid environmental changes (e.g., turbulence, ballistic impact). Some of these systems could benefit from real-time state estimation to enable their full potential. Examples of such systems include blast mitigation strategies, automotive airbag technologies, and hypersonic vehicles. Particular challenges in high-rate state estimation include: 1) complex time varying nonlinearities of system (e.g. noise, uncertainty, and disturbance); 2) rapid environmental changes; 3) requirement of high convergence rate. Here, we propose using a Variable Input Observer (VIO) concept to vary the input space as the event unfolds. When systems experience high-rate dynamics, rapid changes in the system occur. To investigate the VIO's potential, a VIO-based neuro-observer is constructed and studied using experimental data collected from a laboratory impact test. Results demonstrate that the input space is unique to different impact conditions, and that adjusting the input space throughout the dynamic event produces better estimations than using a traditional fixed input space strategy.

Keywords

High-rate dynamics, input space, adaptive observer, neural network, structural health monitoring

Disciplines

Civil Engineering | Dynamics and Dynamical Systems | Structural Engineering | VLSI and Circuits, Embedded and Hardware Systems

Comments

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Variable Input Observer for State Estimation of High-Rate Dynamics

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ABSTRACT

High-rate systems operating in the 10 μs to 10 ms timescale are likely to experience damaging effects due to rapid environmental changes (e.g., turbulence, ballistic impact). Some of these systems could benefit from real-time state estimation to enable their full potential. Examples of such systems include blast mitigation strategies, automotive airbag technologies, and hypersonic vehicles. Particular challenges in high-rate state estimation include: 1) complex time varying nonlinearities of system (e.g. noise, uncertainty, and disturbance); 2) rapid environmental changes; 3) requirement of high convergence rate. Here, we propose using a Variable Input Observer (VIO) concept to vary the input space as the event unfolds. When systems experience high-rate dynamics, rapid changes in the system occur. To investigate the VIO's potential, a VIO-based neuro-observer is constructed and studied using experimental data collected from a laboratory impact test. Results demonstrate that the input space is unique to different impact conditions, and that adjusting the input space throughout the dynamic event produces better estimations than using a traditional fixed input space strategy.

Keywords: High-rate dynamics, input space, adaptive observer, neural network, structural health monitoring

1. INTRODUCTION

High-rate dynamic environments are defined in[1] as environments comprised of high-rate and high-amplitude events. Examples of these systems include active protection systems for civil structures to withstand blast,[2] smart automotive airbag systems,[3,4] and in-flight monitoring and rapid guidance adaptability for space shuttles, aircrafts, and hypersonic airframes.[5,6] The failure of such systems may result in numerous casualties. With the capability of detecting these high-rate events on-time, it is possible to significantly increase safety and minimize damage.

State estimation of systems experiencing high-rate dynamics is a challenging task. Much research in state estimation has been geared towards optimizing methods. Although it is difficult to fully compare estimators, most are designed to perform as fast as possible. Examples of estimators for sensorless induction motors have been studied in Ref.[7,8] which both demonstrated estimators operating in the μs timescales. Induction motors and similar systems are considered fast, but are distinctly different from our systems of interest. High-rate dynamic systems include added complexities such as large uncertainties in the external loads, high levels of nonstationarities and disturbances, and generation of unmodeled dynamics from rapid changes in mechanical configurations. These complexities have been considered individually in Ref.,[9–15] however, they have yet to be considered altogether.

It was noted in Ref.[1] that adaptive observer techniques have a particular promise for the state estimation of complex dynamic systems. However, adaptive observers are generally slower than traditional model-driven observers, and further extensions of current adaptive methods need to be researched in order to broaden their applicability to high-rate systems. As a potential solution, the authors proposed to construct an adaptive observer capable of varying its input space in real time, termed Variable Input Observer (VIO). The concept of the VIO is based on the embedding theorem, where the only input that are used are those that preserve the essential dynamics of the system of interest.[16] Given that the system of interest is highly nonlinear and nonstationary, such inputs will change as a function of time. Input space selection based on the embedding theorem has been proposed before. However, research is limited to selection strategies based on offline

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batch processing, and the input space, once selected, remains fixed. Examples of such work can be found in Refs.[17–20] The authors have proposed an adaptive input space using an online strategy, with applications to structural control.[21–23]

In preliminary work on the VIO,[1] we demonstrated the concept of the VIO using a simple time-delay function. The VIO was compared with an adaptive Luenberger observer, Kalman filter, and fixed-input observer based on the same delay function. The analysis was conducted on data from a simulated two degrees-of-freedom (2-DOF) spring-mass system with varying stiffness. The analysis showed that the VIO was capable of outperforming the other observers in terms of 2-norm error. In this paper, we are building on preliminary results to propose a VIO based on a neuro-observer architecture. The neuro-observer is a single-layer Mexican hat wavelet network. Unlike preliminary work, we simulate the VIO's performance using experimental measurements from a high-rate system.

The rest of the paper is organized as follows. The next section presents the architecture of the VIO and the algorithm use in the online selection of the input space. The subsequent section presents and discusses numerical simulations conducted using experimental data. The last section concludes the paper.

2. ARCHITECTURE OF THE VARIABLE INPUT SPACE OBSERVER

The principle of the VIO is to construct and modify a state estimation's input space in real-time. This is done by embedding a given measurement' time series $\mathbf{y}$ into a vector constructed using a time delay τ and embedding dimension, d, such that

$$\boldsymbol{v}(k) = \begin{bmatrix} y(k) & y(k-\tau) & y(k-2\tau) & \cdots & y(k-(d-1)\tau \end{bmatrix} \tag{1}$$

where $\boldsymbol{v}$ is termed the delay vector, and k is a discrete time step with τ the number of delayed time steps. The proper choice of τ and d are those that only preserve the essential dynamics of the system, therefore optimizing the use of incoming data, minimizing the size of the input space, and optimizing the performance of the estimation function when adaptive. The block diagram in Fig. 1 schematizes the VIO. The mutual information test is used to select τ form the measurements, then the false nearest neighbors test is used to select d, followed by the construction of the input space from $\boldsymbol{v}$. The input space is fed in the state-estimation function, here a neuro-observer, which contains adaptive capabilities based on an estimation error metric.

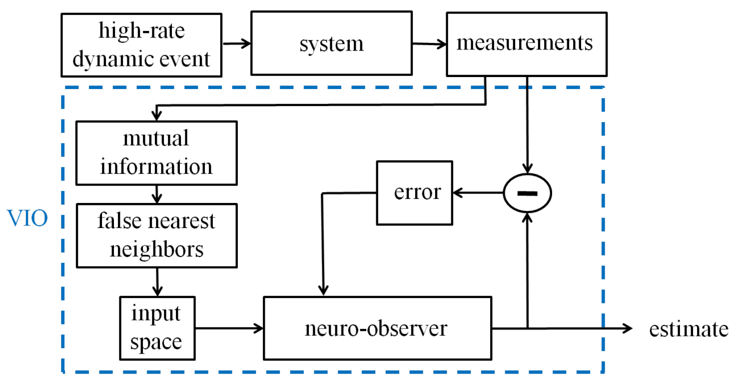

Figure 1: Schematic of the VIO.

2.1 Neuro-Observer

The state estimation function of the VIO is a single-layer wavelet neural network, where the estimated state y_k is written

$$\hat{y}_k = \sum_{j=1}^{h} \gamma_j \phi_j(\boldsymbol{v}) \tag{2}$$

where h represents the number of nodes, γ the nodal weights of node j, and ϕ is the activation function taken as a Mexican hat wavelet

$$\phi(\boldsymbol{\nu}) = \left(1 - \frac{\|\boldsymbol{\nu}-\boldsymbol{\mu}\|_2}{\boldsymbol{\sigma}^2}\right) e^{-\frac{\|\boldsymbol{\nu}-\boldsymbol{\mu}\|_2}{\boldsymbol{\sigma}^2}} \tag{3}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are the wavelet centers and bandwidths, respectively, and $\|\cdot\|_2$ is the 2-norm. The VIO is designed to be capable of sequential adaptive learning. Also, a self-organizing mapping architecture is adopted to minimize the network size,[24] which consists of adding a new network node if a new observation falls outside an Euclidean distance threshold D to the closest available node. When a new node j is added, it is given a weight γ_j initially equal to zero, a center $\boldsymbol{\mu}_j$ at the location of the new observation, and the bandwidth $\boldsymbol{\sigma}_j$ of the newly added wavelet initially set at 1. If no new nodes are added, the network is put in an adaptation mode, where weights and bandwidths are adapted following a back-propagation rule:[16]

$$\begin{aligned} \gamma_j[k+1] &= \gamma_j[k] - \Gamma_{\gamma_j}\phi_j(\boldsymbol{\nu})\tilde{y} \\ \sigma_j[k+1] &= \sigma_j[k] - \Gamma_{\sigma_j}\gamma_j\left(\frac{1}{\sigma_j^5}\mathrm{e}^{\frac{-\|\boldsymbol{\nu}-\mu_j\|^2}{\sigma_j^2}}(4\sigma_j^2\|\boldsymbol{\nu}-\mu_j\|^2 - 2\|\boldsymbol{\nu}-\mu_j\|^4)\right)\tilde{y} \end{aligned} \tag{4}$$

where $\tilde{y}$ is the observation error between the estimated and the measured state and Γ is the learning rate associated with adaptive parameter in subscript.

2.2 Input Space Adaptation

In a traditional observer, the delay vector $\boldsymbol{\nu}$ (Eq. 1) would have fixed values of τ and d. Here, the VIO sequentially inspects the dynamics of a given measurement's time series $\boldsymbol{y}$ and selects values of τ and d such that the essential dynamics of the system is preserved. There are different embedding methods for calculating the appropriate τ and d values. Based on previous work and Cellucci *et al.*,[25] the use of mutual information for calculating τ and the false nearest neighbors algorithm for identifying d have showed great promise and are used herein.

Thus, τ is first obtained using the mutual information (MI) test[26]

$$\mathrm{MI}(x,y) = \sum_{x,y} p(y[k], y[k-\tau]) \log \frac{p(y(k), y(k-\tau))}{p(y(k), p(y(k-\tau)))} \tag{5}$$

where $y(k)$ and $y(k-\tau)$ are discrete observations of the time series, $p(\cdot)$ indicates a probability, and $p(\cdot,\cdot)$ indicates a joint probability. Depending on the value of τ, the information between $y(k)$ and $y(k-\tau)$ can be either relevant or redundant. The optimal value of τ is the value which produces the most relevant information and is determined from the first minimum of the mutual information function.

Second, the false nearest neighbor (FNN) algorithm is used to calculate the optimal embedding dimension, d of the input space from eqn. (1). The algorithm is based on Kennel *et al.*[27] The algorithm calculates the distance between the rth neighboring points of a vector before and after increasing its dimensions. If the distance is greater than some threshold, the point is considered a false neighbor:

$$\left|\frac{R^2_{d+1}(m,r) - R^2_d(m,r)}{R^2_d(m,r)}\right| > R_{tol} \tag{6}$$

where R_{tol} is a user defined threshold and $R_d(m,r)$ and $R_{d+1}(m,r)$ are distance matrices of dimension d and $d+1$. A second condition is added for increased accuracy

$$\frac{R_{d+1}(m)}{R_A} > A_{tol} \tag{7}$$

where

$$R_A^2 = \frac{1}{n}\sum_{m=1}^{n}(y(m)-\bar{y})^2 \tag{8}$$

and A_{tol} is a user defined value. If either expression is true, the points are considered false neighbors. The number of false neighbors as a function of the embedding dimension is used to find an optimal value for d.

3. NUMERICAL SIMULATIONS

The proposed neuro-based VIO was numerically simulated on experimental data obtained from a high-rate system with the purpose to demonstrate the performance of the VIO on realistic data. As a comparison, estimations were also made using a fixed-input space observer with the same neuro-architecture, where the time delay τ and embedding dimension d were kept constant. The values for τ and d that we used were calculated from a parametric study of τ ranging from 1 to 400 and d ranging from 1 to 10. The combination of τ and d that resulted in the lowest 2-norm error was used for the fixed observer. In what follows, we first describe the details of the experiment from which data were acquired and then discuss the simulation results.

3.1 Experimental Setup

Data was gathered from a laboratory experiment of an electronics package on a accelerated drop tower (MTS-66). The electronics package had four high $(g)_n$ shock accelerometers mounted to circuit boards. Only data from two accelerometers (accel 1 and accel 4) were used to create an input-output type scenario for the simulations. The drop tower is accelerated by stiff bungee cables to create an impact event. Fig. 2 illustrates the experiment.

Data was captured using a Precision Filters signal conditioning system coupled with a National Instruments data acquisition (DAQ) system. Analog signal conditioning for the accelerometers was accomplished via a Precision Filter 28000 chassis with 28144A Quad-Channel Wideband Transducer Conditioner in constant voltage excitation mode. A anti-aliasing filter of 204.6 kHz was selected. A high-rate instrumentation system using a National Instruments chassis paired with PXI-6133 acquisition cards sampling at 1 Msa/s.

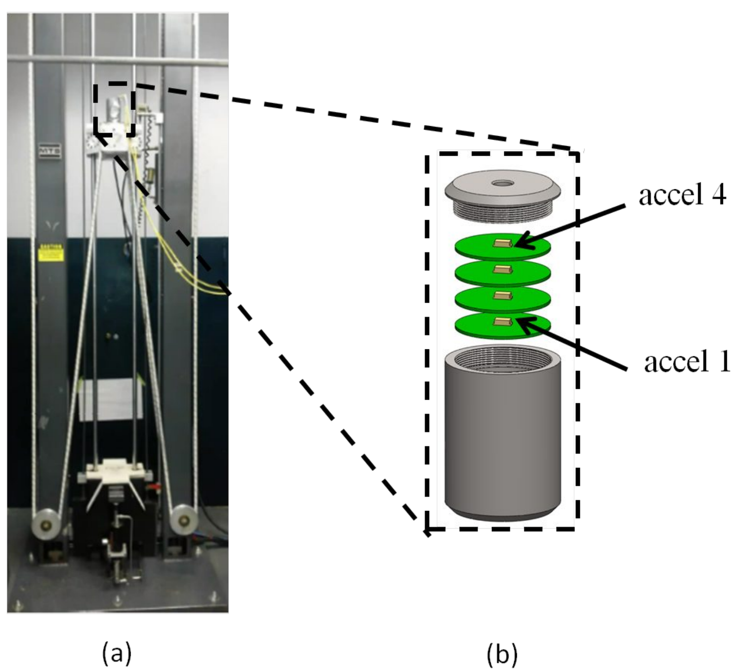

Figure 2: Experimental setup: a) drop tower; b) electronic package.

Due to the event occurring at a very fast rate with large amplitudes in accelerations, the response of the electronics package has the attributes of a high rate dynamics system resulting in large uncertainties on the external loads. Additionally, the large external loads produce nonstationarities in the system possibly through the debonding of the potting material within the inside of the metal housing. Furthermore, there is noise present from cable movement (cable from sensor to DAQ) and rattling of metal interfaces from loss of torque due to impact.

Minimal tuning of the VIO was conducted to analyze its performance and make a comparison with a fixed-input strategy, which used an input space constructed with $\tau = 46$ and $d = 3$. The parameters for the numerical simulation are listed in Table 1.

Table 1: Parameter values for the VIO

Parameter	Value
Γ_γ	0.2
Γ_σ	1
$\sigma(0)$	1
R_{tol}	1
A_{tol}	0.1
Data length	2000
FNN percentage	20

3.2 Simulation Results

The VIO is implemented through a sequential adaptation process as described above. Simulation results are plotted in Fig. 3 and 4, showing the time-series of states and absolute estimation errors, respectively. The data from accel 1 was used to estimate the output of accel 4. Results show that the VIO produces performs well at state estimation from the start, which is confirmed by both the time series of the estimated state and the absolute estimation error. It also converges faster than the fixed input strategy. However, the VIO produces some instability as seen by the large error spikes. These instabilities can be attributed to the fast changes in the input space. Fig. 5 is a plot of the variation in τ and d. Results show that the absolute error in the VIO increases when τ drops abruptly. Sudden changes in the input space, in particular τ, require large changes in other adaptive parameters ($\boldsymbol{\sigma}$ and γ), which produces the oscillatory effect observed in Fig. 3.

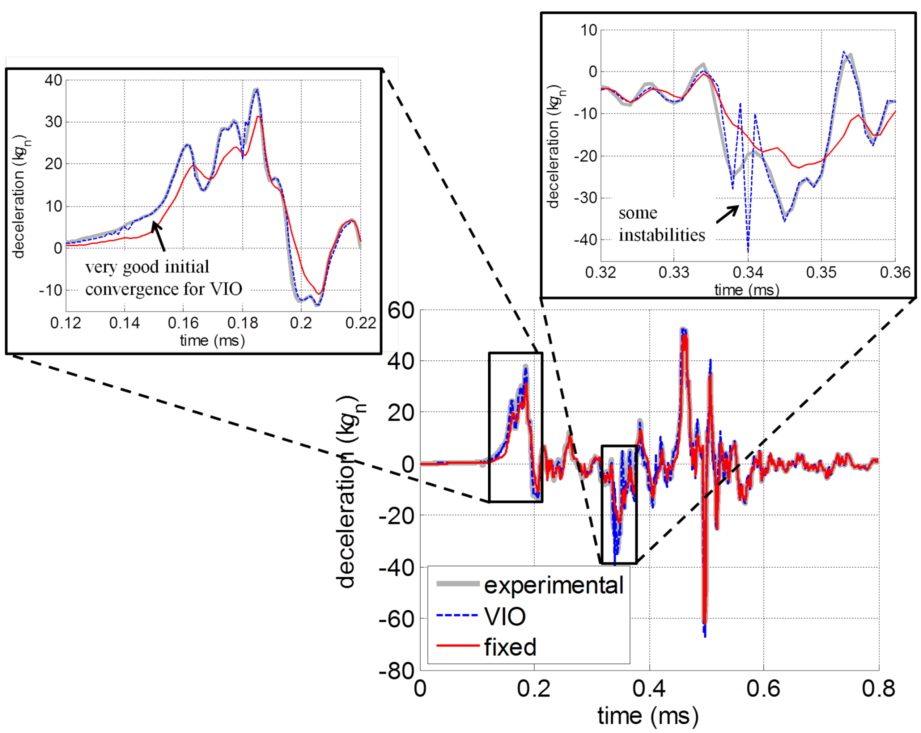

Figure 3: Time series of real and estimated states.

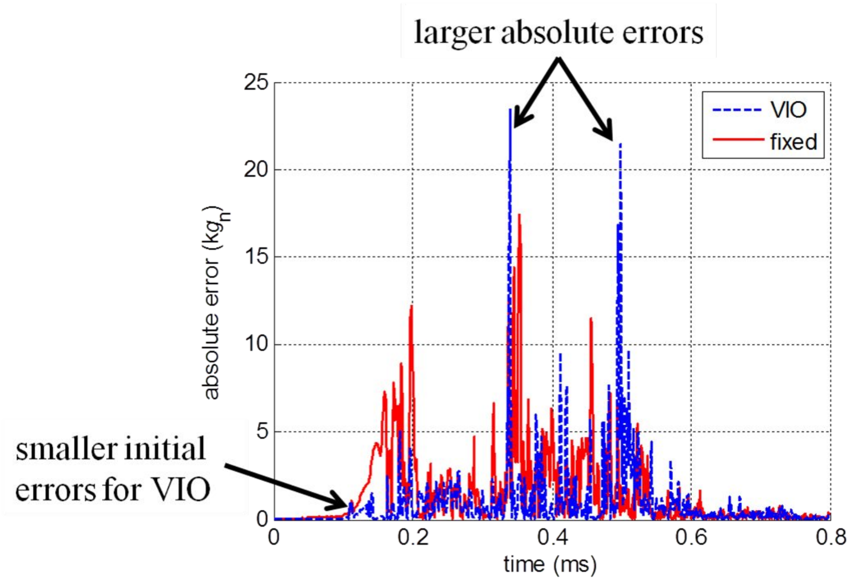

Figure 4: Absolute error of VIO and fixed observer.

The overall performance of the observer can be quantified by two performance metrics. The first performance metric, J_1 is defined as:

$$J_1 = \frac{\|(\hat{\mathbf{y}} - \ddot{\mathbf{x}}_4)\|_2}{\|\ddot{\mathbf{x}}_4\|_2} \tag{9}$$

with $\hat{\mathbf{y}} = \hat{\ddot{\mathbf{x}}}_4$ being the estimation of the measurement of accel 4, $\ddot{\mathbf{x}}_4$ and $\|\cdot\|_2$ being the 2-norm. By representing the error of the estimate by equation 9, the error is normalized to fall within the values of [0,1]. This normalization is is used to compare the VIO's performance with the fixed-input strategy.

The second performance metric J_2 is in terms of the convergence rate. The convergence rate is defined as the time it takes from the start of the impact ($>100\ g_n$) to when the estimation error falls and remains within an error threshold. The error threshold was determined to be 5% of the initial peak and is governed by the variations in the data created by the experimental setup.

The third performance metric J_3 compares the maximum error of the VIO and fixed observer from the start of the simulation to 0.335 ms into the estimation. The metric is reported in % in comparison with the initial peak. The purpose is to compare the initial convergences between the two observers.

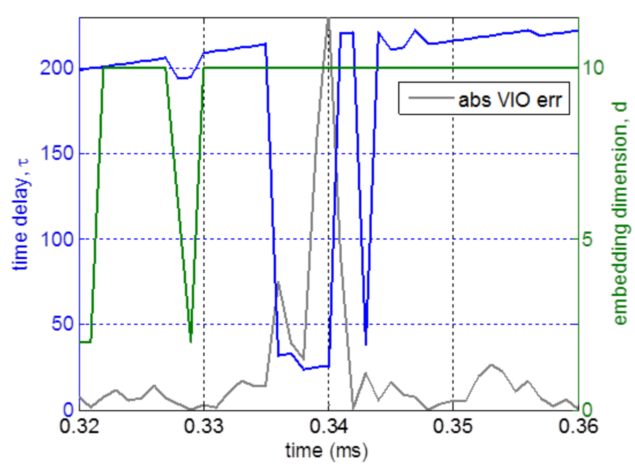

Figure 5: Variation of τ and d through the estimation process.

The comparison of the performance metrics for the VIO and the fixed observer is presented in Table 2. In comparison with the fixed observer, the VIO required less than half the number of nodes for the estimation, which translates to less computation due to the more accurate representation of the system dynamics. The VIO also showed a 22% improvement in terms of J_1. The VIO suffered in the J_2 metric and performed poorer than the fixed observer because of the instabilities that resulted in larger error spikes later in the estimation. In the future we will reduce these instabilities by restricting the rate of change of the input space. In the J_3 metric, we see that the VIO had much better initial convergence from the maximum error only reaching 13% in the front part of the estimation (a 57% improvement as compared with the fixed observer).

Table 2: Performance of VIO versus fixed observer.

observer type	number of nodes	J_1	J_2 (ms)	J_3 (%)
VIO	11	0.186	0.573	13
fixed	27	0.238	0.484	30

4. CONCLUSION

State estimation of systems experiencing high-rate dynamic events is an important area of research which will help pave the way for safer and smarter systems that require real-time observability for instantaneous decision making. The application of this research ranges from civil structures experiencing high-dynamic loads such as blast to hypersonic aerial vehicles encountering foreign objects. In our previous study, we developed the variable input space (VIO) concept, which consists of an estimator capable of adapting its input space in real time. We theorized that it could be a useful tool for systems experiencing high-rate dynamics, due to the system-specific large nonlinearities and nonstationarities that necessitate better adaptability.

In this paper, we demonstrated the VIO on experimental high-rate data obtained through a laboratory experiment, using a neural network architecture for the estimation function. We compared the VIO with a fixed input strategy, using the same estimation function. The comparison showed significant advantaged to the VIO method to the initial estimations and a 22% improvement in overall 2-norm error using less than half the number of nodes. Due to the rapid changes in input space, the VIO suffered from instabilities later in the estimation process which resulted in decreased performance for overall convergence rate. Nevertheless, the initial convergence of the VIO was 57% better than the fixed observer.

Work presented in this paper is the result of a preliminary investigation on the potential of the VIO concept. Future work include the incorporation of smooth transitions for τ and d to improve on the stability, the adaptation of the data length which is used to calculate the input space to provide an enhanced estimation performance, and the pruning of wavelets which no longer add value to the estimation, which could improve the computation speed.

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REFERENCES

[1] J. Hong, S. Laflamme, and J. Dodson, "Variable input observer for structural health monitoring of high-rate systems," *Quantitative Nondestructive Evaluation* **43**, 2016.

[2] H. N. Wadley, K. P. Dharmasena, M. He, R. M. McMeeking, A. G. Evans, T. Bui-Thanh, and R. Radovitzky, "An active concept for limiting injuries caused by air blasts," *International Journal of Impact Engineering* **37**(3), pp. 317–323, 2010.

[3] S.-J. Lee, M.-S. Jang, Y.-G. Kim, and G.-T. Park, "Stereovision-based real-time occupant classification system for advanced airbag systems," *International Journal of Automotive Technology* **12**(3), pp. 425–432, 2011.

[4] "Trw introduces adaptive airbags." http://safety.trw.com. Accessed: 2011-12-06.
[5] J. D. Walker, "From columbia to discovery: Understanding the impact threat to the space shuttle," *International Journal of Impact Engineering* **36**(2), pp. 303–317, 2009.
[6] Y. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Annual Reviews in Control* **32**(2), pp. 229–252, 2008.
[7] Z. Xu, F. Rahman, and D. Xu, "Comparative study of an adaptive sliding observer and an ekf for speed sensor-less dtc ipm synchronous motor drives," in *Power Electronics Specialists Conference*, pp. 2586–2592, IEEE, 2007.
[8] Y. Zhang, Z. Zhao, T. Lu, L. Yuan, W. Xu, and J. Zhu, "A comparative study of luenberger observer, sliding mode observer and extended kalman filter for sensorless vector control of induction motor drives," in *Energy Conversion Congress and Exposition*, pp. 2466–2473, IEEE, 2009.
[9] S. S. Kourehli, A. Bagheri, G. G. Amiri, and M. Ghafory-Ashtiany, "Structural damage detection using incomplete modal data and incomplete static response," *KSCE Journal of Civil Engineering* **17**(1), pp. 216–223, 2013.
[10] G.-L. Tao and Z.-l. Deng, "Convergence of self-tuning riccati equation for systems with unknown parameters and noise variances," in *8th World Congress on Intelligent Control and Automation (WCICA)*, pp. 5732–5736, IEEE, 2010.
[11] R. M. F. Oliveira, E. Ferreira, F. Oliveira, and S. Azevedo, "A study on the convergence of observer-based kinetics estimators in stirred tank bioreactors," *Korean Institute of Chemical Engineers* **6**(6), pp. 367–371, 1994.
[12] K. Stricker, L. Kocher, D. Van Alstine, and G. M. Shaver, "Input observer convergence and robustness: Application to compression ratio estimation," *Control Engineering Practice* **21**(4), pp. 565–582, 2013.
[13] K. Yamada and M. Kobayashi, "A design method for unknown input observer for non-minimum phase systems," in *International Workshop and Conference on Photonics and Nanotechnology*, pp. 1–6, International Society for Optics and Photonics, 2007.
[14] Y. Wang, R. Rajamani, and D. M. Bevly, "Observer design for differentiable lipschitz nonlinear systems with time-varying parameters," in *53rd Annual Conference on Decision and Control (CDC)*, pp. 145–152, IEEE, 2014.
[15] B. K. Kim, W. K. Chung, and K. Ohba, "Design and performance tuning of sliding-mode controller for high-speed and high-accuracy positioning systems in disturbance observer framework," *IEEE Transactions on Industrial Electronics* **56**(10), pp. 3798–3809, 2009.
[16] S. Laflamme, J. E. Slotine, and J. Connor, "Self-organizing input space for control of structures," *Smart Materials and Structures* **21**(11), p. 115015, 2012.
[17] E. Monroig, *Detection of Changes in Dynamical Systems by Nonlinear Time Series Analysis*. PhD thesis, University of Tokyo, 2009.
[18] E. Figueiredo, M. Todd, C. Farrar, and E. Flynn, "Autoregressive modeling with state-space embedding vectors for damage detection under operational variability," *International Journal of Engineering Science* **48**(10), pp. 822–834, 2010.
[19] G. Liu, Z. Mao, M. D. Todd, and Z. Huang, "Damage assessment with state-space embedding strategy and singular value decomposition under stochastic excitation," *Structural Health Monitoring* , pp. 131–142, 2013.
[20] V. Chinde, L. Cao, U. Vaidya, and S. Laflamme, "Damage detection on mesosurfaces using distributed sensor network and spectral diffusion maps," *Measurement Science and Technology* **27**(4), p. 045110, 2016.
[21] L. Cao and S. Laflamme, "Input space-dependent controller for multi-hazard mitigation," *SPIE Smart Structures and Materials + Nondestructive Evaluation and Health Monitoring* , 2016.
[22] L. Cao and S. Laflamme, "Multi-delay controller for multi-hazard mitigation," *American Control Conference (ACC)* , pp. 1851–1856, 2016.
[23] L. Cao and S. Laflamme, "Real-time variable multi delay controller for multi-hazard mitigation," *ASCE Journal of Engineering Mechanics* **Pending review**, 2017.
[24] T. Kohonen, "The self-organizing map," *Neurocomputing* **21**(1–3), pp. 1–6, 1998.
[25] C. J. Cellucci, A. M. Albano, and P. E. Rapp, "Comparative study of embedding methods," *Physical Review E* **67**, pp. 1–13, 2003.
[26] A. Fraser and H. Swinney, "Independent coordinates for strange attractors from mutual information," *Physical Review A* **33**(2), pp. 1134–1140, 1986.
[27] M. Kennel, R. Brown, and H. Abarbanel, "Determining embedding dimension for phase-space reconstruction using a geometrical construction," *Physical Review A* **45**(6), pp. 3403–3411, 1992.